CBF-Based Hierarchical Quadratic Programs with Guaranteed Feasibility for Safety-Critical Systems

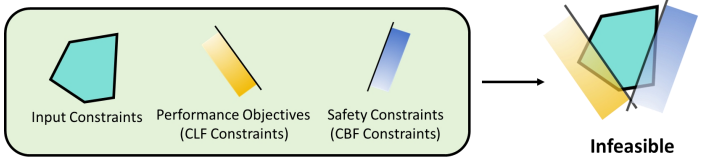
Junjun Xie, Liang Hu, Yunzhe Tan, and Jun Yang

Harbin Institute of Technology, Shenzhen
Loughborough University

Motivation

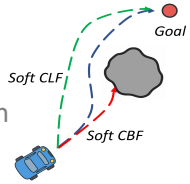
- ***Infeasibility*** under multiple hard constraints

In CLF-CBF-QP-based controller design frameworks, multiple constraints may conflict with each other, making the optimization problem infeasible and preventing the desired controller from being computed.



- ***Performance degradation*** under weighted soft constraints

Soft constraints can ensure feasibility, but may degrade the performance guaranteed by the constraints, even when the system can strictly satisfy the prescribed requirements.



Safety-first CLF-CBF QP

A hierarchical optimization framework

$$[\sim,\sim,\delta_2^*] = \arg\min_{[u,\delta_1,\delta_2]} \ \delta_2^2 \quad \text{s.t.} \ L_f V(x) + L_g V(x) u + \lambda V(x) \le \delta_1,\ L_f h(x) + L_g h(x) u + \gamma h(x) \ge \delta_2,\ u \in \mathcal{U} \quad \text{(Safety)}$$

$$\xrightarrow{\delta_2^*}$$

$$[\sim,\delta_1^*] = \arg\min_{[u,\delta_1]} \ \delta_1^2 \quad \text{s.t.} \ L_f V(x) + L_g V(x) u + \lambda V(x) \le \delta_1,\ L_f h(x) + L_g h(x) u + \gamma h(x) \ge \delta_2^*,\ u \in \mathcal{U} \quad \text{(Performance)}$$

$$\xrightarrow{\delta_1^*}$$

$$u^* = \arg\min_{u} \ u^T H u \quad \text{s.t.} \ L_f V(x) + L_g V(x) u + \lambda V(x) \le \delta_1^*,\ L_f h(x) + L_g h(x) u + \gamma h(x) \ge \delta_2^*,\ u \in \mathcal{U} \quad \text{(Minimal Input)}$$

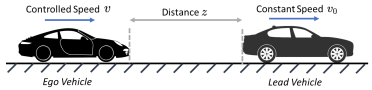
$$\rightarrow u^*$$

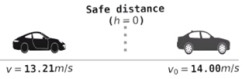
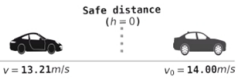
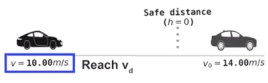
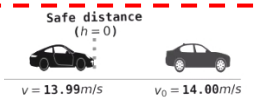
Core idea:

compute the minimum relaxation sequentially according to constraint priorities.

Case Study: Adaptive Cruise Control

Performance objective: **desired speed** v_d
Safety requirement: **distance** $z > 0$

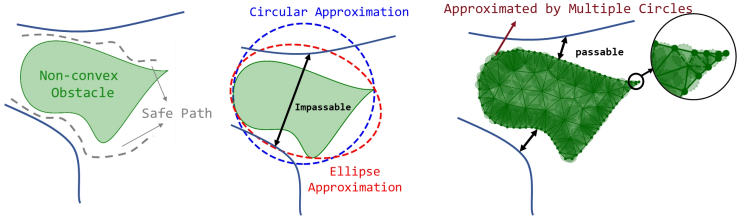


Case	CLF-CBF QP	Optimal-decay CLF-CBF QP	Safety-first CLF-CBF QP
1			
			reach target speed fastest
Case 1: Lead vehicle speed 14 m/s *Ego target speed* ***10 m/s***			
2 *Case 2: Lead vehicle speed* 14 m/s *Ego target speed* ***20 m/s***	***infeasible***	 ***collision***	 ***safe***

Safe Navigation

Constrained Delaunay Triangulation (CDT)-based Approximation

Our method can handle multiple constraints. Therefore, we approximate irregular obstacles with multiple circles to improve performance in narrow and cluttered environments.



original *common approach* ***CDT-based approximation***

Real-world Experiments

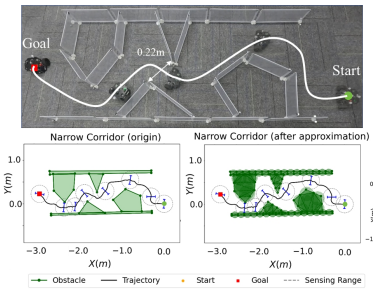


Pipeline

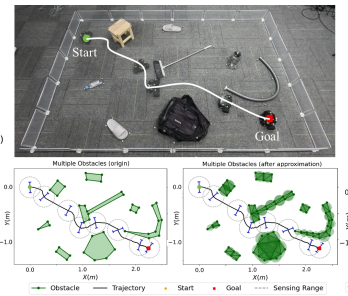
COMPUTATION TIME UNDER DIFFERENT NUMBERS OF CIRCLES

# of circles	1	5	10	20
CLF-CBF QP	1.7ms	2.1ms	2.4ms	5.4ms
Optimal-decay CLF-CBF QP	1.9ms	2.2ms	2.5ms	5.8ms
Safety-first CLF-CBF QP	3.7ms	7.5ms	12.9ms	23.5ms

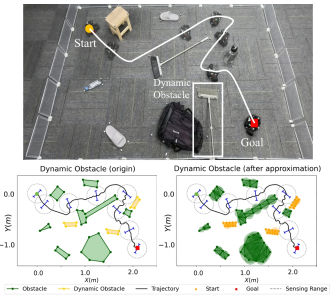
*Safety-first CLF-CBF QP remains real-time with multiple CBF constraints (~80 Hz with 10 circular CBFs).



Narrow Corridor



Multiple Obstacles



Dynamic Obstacle

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哈爾濱工業大學（深圳） HARBIN INSTITUTE OF TECHNOLOGY, SHENZHEN

CLASS-LAB Control and Learning for Autonomous SystemS Lab

Loughborough University